DESIGN OF HEAT EXCHANGERS WITH A CROSSFLOW OF HEAT CARRIERS

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The performance of crossflow heat exchangers with a nonuniform velocity distribution in the gaseous (air) heat carrier is analyzed. Design formulas are derived for determining the thermal load under conditions of a uniform and a nonuniform velocity distribution respectively.

The need for a study of crossflow heat exchangers with mixing in the liquid and with a nonuniform velocity distribution in the gas (air) arises from the wide range of their technical applications (e.g., in radiators of various type vehicles).

Many studies have dealt with methods of designing heat exchangers, but only in a few of them was the crossflow case considered. In [1] was shown a mathematical model with an analytical solution for a heat exchanger with crossflow but without mixing in the heat carriers. In [2] the analytical solution for this case is given but not in complete form. There the calculations of crossflow heat exchangers with mixing in one but without mixing in the other heat carrier yield sometimes appreciable deviations from test data. The effect of a nonuniform velocity in one of the heat carriers on the performance of the heat exchanger is not considered in [1] and [2]. The reduction in the heat dissipating capacity of a heat exchanger with a nonuniform velocity distribution (nonuniform flow rates in the tubes) is determined in [3] but only for the case of a parallel flow.

We will consider the case of crossflow where one of the heat carriers (the liquid) is mixing, i.e., has the same weighted mean temperature over a section while the other heat carrier (the gas stream) is not.

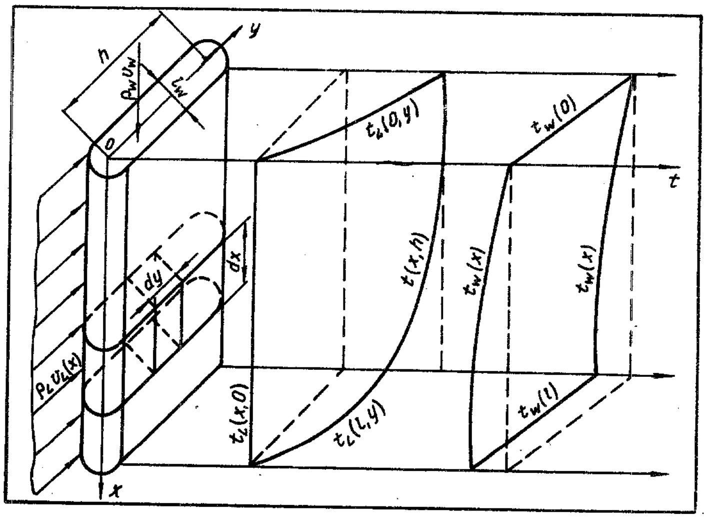

Fig. 1. Schematic diagram representing the temperature variations in the heat carriers.

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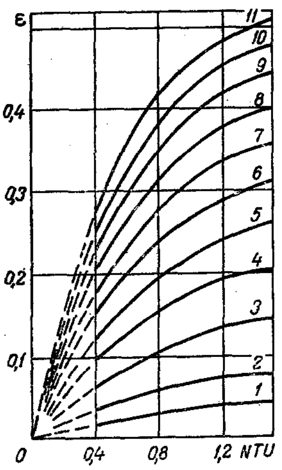

Fig. 2

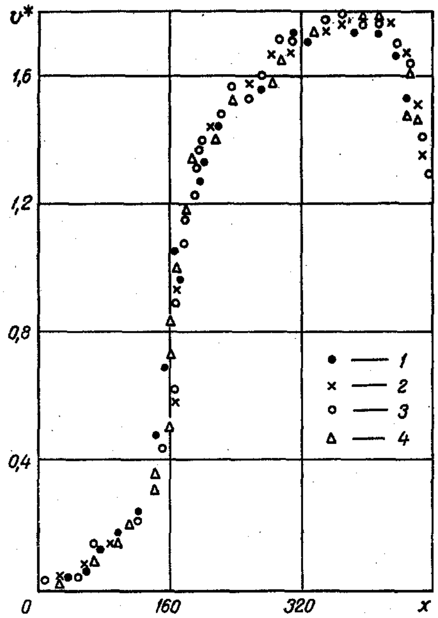

Fig. 3

Fig. 2. Curves of $\varepsilon = f(NTU, W_G/W_L)$: 1) $W_G/W_L = 0.05$; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.4; 6) 0.5; 7) 0.6; 8) 0.7; 9) 0.8; 10) 0.9; 11) 1.0.

Fig. 3. Distribution of relative velocity $v^* = v_G/\bar{v}_G$ along the height of a heat exchanger: 1) $\rho_G \bar{v}_G = 5$ kg/m² · sec; 2) 8 kg /m² · sec; 3) 11 kg/m² · sec; 4) 14 kg/m² · sec. Height x (mm).

We will cut the heat exchanger with parallel planes perpendicular to its front surface and passing through the axes of tubes adjoining the front surface. The cooling area contained between the cutting planes consists of the finned tube surfaces.

In order to simplify the calculations, we will replace a finned tube of the heat exchanger by a rectangular tube having the same surface area as well as the same active section area on the gas side and on the liquid side respectively as a real finned tube adjoining the front surface.

We will also adopt the procedure – classical by now – of averaging the flow characteristics and thus also the heat transfer coefficients and the heat transmittivities over the surface or part of it.

In our arrangment shown in Fig. 1 the quantity of heat passing through a surface element is described by the following equation in relative coordinates:

$$dQ = kF\,[t_L\,(x^*,\ y^*) - t_G(x^*,\ y^*)]\,dx^*dy^*. \tag{1}$$

The temperature of the liquid will drop here by $(\partial t_L(x^*, y^*)/\partial x^*)dx^*$, while the gas temperature will rise by $(\partial t_G(x^*, y^*)/\partial y^*)dy^*$.

The quantity of heat which the liquid loses and the gas gains is

$$dQ = W_G v^* dx^* \frac{\partial t_G (x^*,\ y^*)}{\partial y^*} dy^*,$$

$$dQ = -W_G dy^* \frac{\partial t_L (x^*,\ y^*)}{\partial x^*} dx^*. \tag{2}$$

Equating (2) and (3) to (1) and assuming that

$$t_L (x^*,\ y^*) = t_L (x^*), \tag{3}$$

in accordance with the stipulation that the liquid mixes, we obtain two equations:

$$\frac{\partial t_G (x^*,\ y^*)}{\partial y^*} + \frac{NTU}{v^*}\, t_G(x^*,\ y^*) = \frac{NTU}{v^*} t_L (x^*), \tag{4}$$

$$\frac{dt_L (x^*)}{dx^*} = NTU \frac{W_G}{W_L} [t_L (x^*) - t_G(x^*,\ y^*)]. \tag{5}$$

TABLE 1. Comparison between Values of Q Calculated by (11), (12) and Tested, with a Uniform and with a Nonuniform Velocity Distribution in the Gas (Air) Stream

W_L W/deg	W_G, W/deg	$\rho_G^v G$, kg/m^2·sec	Q with a uniform velocity distribution W	Nonuniform velocity distribution		
				Q_{test}, W	Q_{calc}, W	discrepancy, %
3240	967	14,0	44000	36700	35000	4,75
3240	760	11,0	40200	33800	31950	5,7
3240	553	8,0	34000	28650	27200	5,1
3240	345	5,0	25500	20500	19500	4,83
2790	967	14,0	39800	35000	33200	5,0
2790	760	11,0	36000	31400	29700	5,5
2790	553	8,0	30000	26100	24600	5,8
2790	345	5,0	23400	21000	19800	5,8

Solving Eq. (4) with the initial conditions $y^* = 0$, $t_G(x^*, y^*) = t_G(x^*, 0)$, i.e., at the gas temperature at the entrance to the heat exchanger, we find the temperature drop between the heat carriers along the surface of the fictitious tube:

$$t_L(x^*) - t_G(x^*, y^*) = \exp\left(-\frac{NTU}{v^*} y^*\right)[t_L(x^*) - t_G(x^*, 0)]. \tag{6}$$

Inserting (6) into (5) and solving the resultant equation with $t_G(x^*, 0) = \text{const}$, with the initial temperature drop between the heat carriers at the entrance equal to $t_L(0) - t_G(x^*, 0) = \Delta t_0$, we finally have

$$t_L(x^*) - t_G(x^*, y^*) = \Delta t_0 \exp\left[-\frac{NTU}{v^*} y^* - \int_0^{x^*} NTU \frac{W_G}{W_L} \exp\left(-\frac{NTU}{v^*} y^*\right) dx^*\right]. \tag{7}$$

Inserting (7) into (1) and integrating over x^*, y^* will yield the heat dissipating capacity (the thermal load) of one fictitious tube:

$$Q = \Delta t_0 W_G \int_0^1 \int_0^1 NTU \exp\left[-\frac{NTU}{v^*} y^* - \int_0^{x^*} NTU \frac{W_G}{W_L} \exp\left(-\frac{NTU}{v^*} y^*\right) dx^*\right] dx^* dy^*. \tag{8}$$

When the weighted velocity of air is uniform along the entire tube (or a tube segment) and thus NTU = const with $v^* = 1$, Eq. (8) then becomes

$$Q = \Delta t_0 W_L \int_0^1 \left\{1 - \exp\left[-NTU \frac{W_G}{W_L} \exp(-NTU y^*)\right]\right\} dy^*. \tag{9}$$

Changing variables under the integral in (9)

$$-NTU \frac{W_G}{W_L} \exp(-NTU y^*) = \xi$$

and considering that $Q = \Delta t_L W_L$, we obtain with $\Delta t_L / \Delta t_0 = \varepsilon$:

$$\varepsilon = \frac{\Delta t_L}{\Delta t_0} = 1 - \frac{1}{NTU} \int_{-NTU \frac{W_G}{W_L} \exp(-NTU)}^{-NTU \frac{W_G}{W_L}} \frac{\exp \xi}{\xi} d\xi. \tag{10}$$

Expression (10) will now be rewritten using the symbols for integral exponential functions:

$$\varepsilon = 1 - \frac{\mathrm{Ei}\left(-NTU \frac{W_G}{W_L}\right) - \mathrm{Ei}\left[-NTU \frac{W_G}{W_L} \exp(-NTU)\right]}{NTU}. \tag{11}$$

In Fig. 2 are shown the values of ε within the ranges of NTU and W_L/W_W which are most typical of heat exchangers for various vehicles.

The heat dissipating capacity or the thermal load of a device with a given nonuniform velocity distribution in the gas (air) stream can be determined from formula (8). Within sufficient accuracy for all practical purposes, however, the formula for the thermal load

$$Q = \varepsilon \Delta t_0 W_L \tag{12}$$

with (11) applies to equal frontal surface segments, and the weighted velocity of the gas before reaching those frontal segments is calculated conventionally on the basis of a rectangular area equivalent to the given section area.

After Q has been determined for 1, 2,3, . . . , n frontal surface segments, the thermal load of the entire heat exchanger is represented as the sum

$$\sum Q = Q_1 + Q_2 + Q_3 + \cdots + Q_n. \tag{13}$$

For illustration, we show in Fig. 3 the distributions of relative velocity in the air stream ahead of the radiator front in some typical vehicles.

In Table 1 are shown tested and calculated values of the heat dissipating capacity of a radiator at various mean velocities of air ahead of the radiator front uniformly and nonuniformly distributed (according to Fig. 3).

The data in Table 1 indicate that a nonuniform velocity distribution in the gas (air) may considerably reduce (by up to 16% in our case) the heat exchanger performance, the difference between calculated and measured values not exceeding 5.8% here.

It must be emphasized that, when NTU = 1, formula (10) yields values for the thermal efficiency of a heat exchanger with a uniform temperature distribution in the heat carriers which differ appreciably from the data in [2]. For instance: $\varepsilon = 0.517$ according to (11) and 0.540 according to [2] when $W_G/W_L = 1$ and NTU = 1.5, $\varepsilon = 0.431$ according to (11) and 0.630 according to [2] when $W_G/W_L = 1$ and NTU = 5.

The preceding analysis shows that in the design of heat exchangers one should take into account the features of their makeup and calculate their thermal load on the basis of a nonuniform velocity distribution in the gaseous heat carrier. Formulas (8), (11), (12), and (13) can be used in this case. When the velocity distribution is uniform, heat exchanger calculations by formulas (11) and (12) are quite simple.

NOTATION

$x^* = x/l$, $y^* = y/h$ are the relative coordinates;
f_L is the area of active section in the liquid stream;
l_L is the width of effective section in the liquid stream;
$h = f_L/l_L$ is the equivalent depth of tube;
l is the length of tube;
l_G is the frontal tubing pitch;
$\rho_G v_G$, $\rho_L v_L$ are the weighted velocity of the gas and of the liquid respectively;
$\underline{v}_G$ is the local gas velocity;
$\overline{V}_G$ is the mean gas velocity along a tube;
$v^* = v_G/\overline{v}_G$ is the relative local gas velocity;
k is the heat-transfer coefficient;
F is the surface area of a finned tube on the gas side of the heat transfer;
c_G, c_L are the specific heat of the gas and of the liquid respectively;
W_G, W_L are the water equivalent of the gas and of the liquid respectively;
$NTU = kF/W_G$ is the number of heat transfer units.

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